Myths about Gravity and Tides

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Introduction

The idea for this paper came as I was having a coffee in a beach-front cafe in North Miami Beach, watching the tide, and perusing a newspaper. Thanks to a small-time serendipity, I chanced upon the following piece of local news*: "During a full moon, the moon has a higher gravitational pull, creating a higher tide. Miami Beach psychiatrist Arnold Lieber says that pull affects oceans and people in a similar way, since the human body is mostly water."* [1]

Obviously, the above quoted text is another wonderful example of bad physics [2]. In reality, the Moon does not pull any harder during the full Moon, and a water content of a human body is entirely irrelevant to a question of tidal effect on humans. Yet a subject of tidal effects on small bodies of water comes up quite often, as evidenced by two additional examples.

"Are there ocean tides in fresh water or just the oceans?" is the question that I found in the "Ask Marilyn" column. *"There are tides everywhere on Earth, including not just oceans and lakes but also ground we stand on (which is factor in earthquakes) and the atmosphere we breathe. If you stood still long enough, there would even be tides in your tummy"*, answered Marilyn vos Savant. [3]

In a similar spirit, an astronomy teacher asserted in my local newspaper*: "Here in Southern Illinois as in much of the world, we experience two periods of high and low tides every day. (...) The tidal forces act on all bodies of water, not just oceans (...) At perigee (...) the moon's gravitational tug on Earth is at its maximum and so the waters of the world rise and fall to their maximum extent."* [4]

While statements in both examples seem plausible, in reality tidal effect on small bodies of water of the size of reader's tummy or lakes in Southern Illinois is negligible and therefore impossible to observe.

One should realize that the tidal effect is caused not only by the Moon, but also by the Sun. If the Moon had not existed, there still would have been ocean tides, but not as high, since

the Sun contributes only about 30% of the tidal effect. Furthermore, and more importantly, it is not the magnitude of the gravitational tug *per se* that is responsible for a tidal mechanism, but rather a subtle difference in the gravitational tug on water at various parts of a basin.

Let us first discuss the magnitude of the gravitational tug. The center of Earth is at the average distance $d_s = 1.496 \times 10^{11}$ m from the center of the Sun. The gravitational pull of the Sun on 1 kg mass at the distance d_s is

$$a_s = \frac{GM_s}{d_s^2} = 6.04 \times 10^{-4} \, g \qquad (1)$$

where $g = 9.81$ N/kg. Similar calculation for the Moon, an average distance $d_m = 3.84 \times 10^8$ m away, yields the following value for gravitational pull of the Moon on a unit mass at the center of Earth

$$a_m = \frac{GM_m}{d_m^2} = 3.39 \times 10^{-6} \, g \qquad (2)$$

Comparing Eq.(1) and Eq.(2), we see that the Sun's gravitational pull per unit mass is about 178 times stronger than the Moon's gravitational pull, hardly a surprising result as Earth orbits the Sun, not the Moon. (Nevertheless, when I ask my students what pulls harder on Earth, the Moon or the Sun, they invariably choose the Moon...) Specifically, Earth's, Sun's and Moon's average gravitational pulls on a 80 kg person on the surface of Earth are about 785 N, 0.47 N, and 0.0027 N, respectively. Now, since Earth is in a free fall about Earth-Moon center of mass, which in turn is in a free fall about the Sun-Earth-Moon center of mass (located inside the Sun), no scale would register the last two pulls. To better see it, remember that if you were standing on a scale inside a freely-falling elevator or orbiting space shuttle, the reading on the scale would be zero, and you would experience apparent weightlessness.

The Source of Tides

Let us now discuss the difference in the gravitational tug on water at various parts of a basin. The gravitational pull of the Sun on 1 kg mass on the far side of Earth, a distance of $R = 6.37 \times 10^6$ m further away from Earth's center, is by the amount of Δa_s smaller than a_s, i.e.

$$\Delta a_s = \frac{GM_s}{d_s^2} - \frac{GM_s}{(d_s + R)^2} = \frac{2GM_s R}{d_s^3} = a_s \frac{2R}{d_s} = 0.515 \times 10^{-7} \, g \qquad (3)$$

This is known as a differential pull. Note that in the second step in Eq.(3) we neglected corrections from higher order R/d_s terms. A similar calculation shows that the gravitational pull of the Sun per 1 kg mass at the point on Earth closest to the Sun is in turn greater than a_s, also by the amount Δa_s. This result could be also obtained by taking the differential of the gravitational force equation. Note that Δa_s is inversely proportional to the *cube* of the distance.

To see how tides come about, let us first ignore the existence of the Moon, and suppose the whole Earth is covered with ocean of equal depth everywhere; there is no dry land. The Earth is simply in free fall towards the Sun.

Consider the point C on Earth closest to the Sun and the point F on a far side of Earth. The Sun pulls harder on a unit mass at the point C, not as hard on a unit mass at Earth center O, and weaker yet on a unit mass at point F. The acceleration a_s of Earth as a whole in free fall towards the Sun is determined by the gravitational pull of the Sun on Earth's center. Hence the unit mass at C has a tendency to accelerate towards the Sun with acceleration $a_s + \Delta a_s$, i.e. more than the center of Earth, while a mass at the far side F has a tendency to accelerate towards the Sun with acceleration $a_s - \Delta a_s$, i.e. to lag behind the center of Earth.

This difference in Solar gravitational pulls would have lead to a disintegration of Earth, had Earth's own gravity been to weak to hold Earth together. To an observer on Earth it would have looked like rocks at point C and F were lifted away from the surface of Earth. Fortunately, Earth's own gravity pulls the unit masses at C and F towards Earth center O with a gravitational pull per unit mass equal to g. The combined effect is that if you drop a rock at the point C or F, rock's acceleration towards the center of Earth is $g - \Delta a_s$.

Another important part of the tidal effect is due to the fact that the Sun is at a finite distance from the Earth, and therefore Sun's pull at the point L (halfway from C towards F along Earth's surface) is not exactly parallel to the Sun's pull at the center O, but has a *sin* α component toward the center of the Earth, where α is the angle made between the line from the Sun to the center of the Earth O and the line from the Sun to the point L, i.e. *tan* $\alpha = R/d_s$. As a result, a rock dropped at the point L accelerates towards the center of Earth at the rate of $g + \Delta a_s/2$.

From the point of view of a hypothetical observer located at the center of the Earth, it appears that Earth's gravitational pull on a rock at C and F is reduced by the amount of Δa_s, while the pull at the point L is increased by $\Delta a_s/2$. In other words, while Earth's own gravity pulls the mass m down with the force mg everywhere on Earth's surface, there appears to be another force, the tidal force, pulling the mass m at points C and F up with magnitude $m\Delta a_s$, and down at point L with magnitude $m\Delta a_s/2$, see Figure 1. This is the source of the tidal mechanism, and the origin of the water bulge at C and F. (The tidal mechanism is nicely illustrated in conceptual physics textbooks [5,6].)
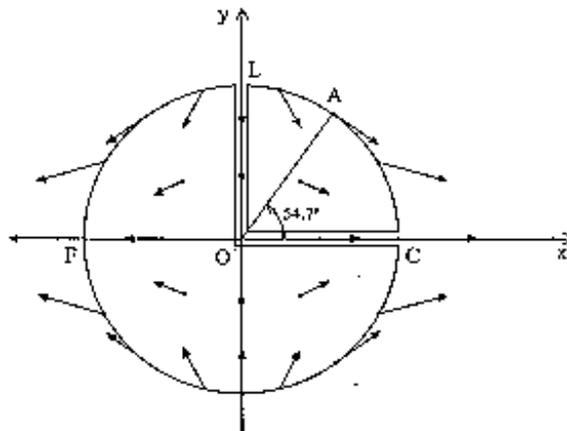
Figure 1. The tidal mechanism, see text. The x-axis points toward the Sun. The Earth is in a free fall toward the Sun. The resulting tidal force causes high tides at the point C closest to the Sun and point F furthest from the Sun, and a low tide at the point L. Note that at the point A the tidal force is tangent to the surface of the Earth.

The lunar tidal effect is calculated in an analogous way. Again, one has to realize that Earth is in a free fall towards the Earth-Moon center of mass. The difference between the Moon's pull on a unit mass at the center of Earth and at the closest/farthest point on Earth is

$$\Delta a_m = a_m \frac{2R}{d_m} = 1.13 \times 10^{-7} g \tag{4}$$

While the solar gravitational pull is 178 times stronger than lunar gravitational pull, the tidal pull is proportional to the inverse <u>cube</u> of the distance, and the ratio of d_s/d_m is 390, thus making the solar tidal pull only $178/390 = 0.46$ of the lunar tidal pull, $\Delta a_s = 0.46\ \Delta a_m$. This could be also seen by a direct comparison of values in Eq.(2) and Eq.(4). Since $\Delta a_m/(\Delta a_m + \Delta a_s) = 69\%$, we see that it is the Moon that dominates the tidal mechanism. But if the Moon had been only twice as far from Earth as it is now, the Moon's tidal pull would have decreased 8 times and become 4 times weaker than the Sun's tidal pull.

When the Sun, Earth and the Moon all lie along a straight line, as at new and full Moon, the Sun's and the Moon's tidal forces pull in the same direction and cause high tides to be higher than average, and low tides to be lower than average. These stronger tides are called spring tides. With the Moon in first or last quarter, the tidal force of the Moon acts in a direction perpendicular to the Sun's tidal force. This makes the tides smaller than average, and they are called neap tides.

First Approximation

Let us first consider the tidal action of the Moon on a hypothetical ocean uniformly covering the whole Earth. For simplicity, I'll neglect effects related to Earth's rotation, as they do not affect our conclusions. The water bulge takes a shape of a football (spheroid or ellipsoid of revolution), and for a not rotating Earth the major axis of the football would point towards the Moon. All water on the surface of the water bulge has equal potential energy, i.e. the water surface is an equipotential. As the Earth is in a free fall, and the water bulge is shaped by the tidal force and Earth's gravity, the equpotential in question in addition to Earth's gravity involves a tidal potential, traditionally called W_2 in literature [7].

At a low-tide point L on the water bulge, the tidal force is directed into Earth, while at a high-tide points C and F, the lunar tidal force is directed up, c.f. Figure 1. If we choose the reference frame with the origin at the center of Earth, the x-axis toward the Moon, and y-axis toward the low-tide point L, c.f. Figure 1, then the Moon's tidal force per unit mass anywhere inside the Earth could be parametrized as

$$\Delta a_{m,x} = a_m \frac{2x}{d_m}$$

$$\Delta a_{m,y} = -a_m \frac{y}{d_m} \tag{5}$$

where a_m is given by Eq.(2). It is perhaps interesting to note that at the point labeled A in Figure 1, the tidal force is purely tangential to Earth's surface. A straightforward calculation shows that the point A is located at the angle $\theta = 54.7°$ away from the x-axis.

Moving the unit mass of water from the low-tide point L to the point C requires a positive work by a tidal force. To calculate this work Newton considered two imaginary wells, one running from the low-tide point L to the center of Earth, and the other from the high-tide point C to the center O. When the unit mass is moved from a low tide point L to the center O to the high-tide point C, the average tidal force along the displacement in the tunnel LO is $\Delta a_{av} = 0.25\Delta a_m$, and in the tunnel OC it is $\Delta a_{av} = 0.50\Delta a_m$, where Δa_m is given by Eq.(4). Hence, the tidal force does a positive work equal to $0.25\Delta a_m R + 0.50\ \Delta a_m R = 0.75\Delta a_m R$, and the tidal potential energy per unit mass decreases by the amount

$$\Delta W_2 = -0.75\ \Delta a_m R \tag{6}$$

This decrease in the tidal gravitational potential must be compensated by the increase of the gravitational potential caused by a rise Δh_m of the water surface with respect to center of Earth, if the water surface is to remain an equipotential. Hence we have

$$g\Delta h_m - 0.75\ \Delta a_m R = 0 \tag{7}$$

Solving for Δh_m, we obtain the value of 0.54 m for the rise of the water level in a homogenous ocean caused by the lunar tidal force. A similar calculation for the solar tidal force yields $\Delta h_s = 0.46\times\Delta h_m = 0.25$ m. Thus during a spring tide one could expect the tidal rise of the water level by $\Delta h = \Delta h_m + \Delta h_s = 0.79$ m.

Mathematical Digression About the Tidal Potential

Results obtained above could be more formally arrived at beginning with a concept of the tidal potential. To this end we need to recognize that the tidal force per unit mass given by Eq.(5) could be written as a gradient of tidal potential energy per unit mass,

$$\Delta a_{m,x} = -\frac{\partial}{\partial x} W_2$$

$$\Delta a_{m,y} = -\frac{\partial}{\partial y} W_2 \tag{8}$$

where the tidal potential is obviously

$$W_2(x, y) = -\frac{GM_m}{d_m^3}(x^2 - \frac{1}{2} y^2) \tag{9}$$

It is convenient to use polar coordinates, $x = r\,cos\,\theta$, $y = r\,sin\,\theta$, and rewrite the tidal potential as

$$W_2(r,\ \theta) = -\frac{GM_m}{d_m^3} r^2 P_2(cos\ \theta) \qquad (10)$$

where P_2 is the Legendre polynomial, $P_2(z) = \frac{1}{2}(3z^2 - 1)$. The main result given by Eq.(6) now simplifies to

$$\Delta W_2 = W_2(R,0°) - W_2(R,90°) = -0.75\,\Delta a_m R \qquad (11)$$

Real Tides

Since in reality Earth rotates and is not uniformly covered with an ocean of a constant depth, analysis of ocean tides is very difficult. Oceans have complicated shapes, varying depths and floor configurations. Shorelines are quite complex. All these factors contribute to unusual local variations of ocean tides [8]. In some places there is only one tide per day. Large basins open to ocean (sounds, gulfs, bays) may exhibit an enormous resonant behavior. There are places where there are no observable tides, and other, where the water rises as much as 51 ft (Minas Basin, Bay of Fundy). In other, there is little tide in a sense of a rise and fall, but strong currents flow periodically back and forth. The high water may not occur when the Moon is overhead, i.e. tides are not necessarily in sync with the differential forces, but may lag behind by several hours. Duration of a high and low water is also affected by local conditions. But for a given location the high water will always lag the passage of the Moon by a fixed amount of hours (this is known as "the establishment of the port" [9].)

What about the tides in smaller bodies of water? The symbol R in Eq.(4) is now replaced by Δd, the difference between a distance to the center of the Moon from the closest and furthest point of the water basin under consideration.

For Great Lakes, the difference in distance from the Moon to various points on a surface of the water is much smaller than radius of the Earth, but tides of amplitude of about two inches could be still observed. But local lakes are so small that all points on the surface of the water are practically at the same distance from the Moon, hence no observable tides exist. Likewise there are no tides in a swimming pool, a bathtub, cup of coffee, or a tummy. Indeed, for a shallow puddle of some 20 m in diameter, the value of Δd due to curvature of Earth is $\Delta d = 1.3 \times 10^{-3}$ m, and Eq.(4) yields $\Delta a_m = 10^{-17}$ g, an unimaginably small value. Hence, no tides in puddles.

As already mentioned above, not only water, but also atmosphere and land are subject to a tidal action and hence experience tidal effects. Because continents are much more rigid than oceans, the effect is much smaller. Nevertheless, parts of continent may rise and fall as much as 0.40m (16 inches) when the Moon passes overhead. So a swimming pool with the water in it may rise few inches, but because the entire land area moves up and down together, we don't readily observe this effect.

Negligible and Small Effects

It is perhaps interesting to estimate the tidal stretch of a human body. Assuming $\Delta d = H = 1.7$ m, one obtains $\Delta a_m = 3 \times 10^{-13}$ N/kg for the Moon's tidal contribution, and at a high tide the combined effect of the Moon and the Sun yields $\Delta a = \Delta a_m + \Delta a_s = 4 \times 10^{-13}$ N/kg. Note that a tidal effect of this magnitude could be alternatively created while holding a pea (mass 1.5 g) some 0.5 m above one's head. In any case, an average tidal tension on a 80 kg person is about 1.6×10^{-11} N. Assuming average cross section area of human skeleton to be 1 cm^2 = 1×10^{-4} m^2, the resulting tidal tensile stress is some 1.6×10^{-7} Pa. Further, taking the value of effective elastic modulus of human skeleton as $E = 1 \times 10^9$ N/m^2, we conclude that the strain is $\Delta H / H = 10^{-16}$, i.e. the expected change in person's height is million times smaller than a size of atom! So much for psychiatrist's "explanation" quoted in the introduction. On the other hand, a calculation of the strain due to a stress produced by the body's own weight yields a reasonable result $\Delta H / H = 1 \times 10^{-2}$.

A related and interesting problem came up as another question in "Ask Marilyn" column. *"We know that the rise and fall of the ocean tides are caused by the gravitational pull of the Moon as it revolves around Earth. Would this same gravitational pull affect the exact weight of a solid object?"* [10]

"Yes", answered Marilyn vos Savant, and this is, in principle, a correct answer. But how big is this effect? How much weight would you lose during a full Moon?

During the full Moon (as during the new Moon), a rock dropped at the point A would fall with acceleration whose component towards the center of Earth is $g = 9.81$ m/s^2. For a 80 kg person, the weight at point A is $W_A = mg = 785$ N or 176 lb. But at the points C and F, the person would fall towards the ground with acceleration

$$g' = g - \Delta g, \tag{12}$$

while at point L this acceleration would be

$$g' = g + \Delta g / 2, \tag{13}$$

where

$$\Delta g = \Delta a_m + \Delta a_s = 1.65 \times 10^{-7} \, g. \tag{14}$$

Therefore the difference between downward accelerations at points C and L is $3/2 \times \Delta g = 2.4 \times 10^{-7}$ g, and an apparent weight of a 80 kg person at points C and F would be by the amount $\Delta W = 2.0 \times 10^{-4}$ N $= 0.4 \times 10^{-4}$ lb smaller than at point L. If the scale were calibrated in kilograms, the reading would be 20 mg (milligram) less. Hence during the spring tide your weight oscillates with a period of 12.5 hours, and with the full Moon overhead (or during the lunch time) you weigh some 2.5×10^{-5} percent less than some 6 hours earlier or later.

Is it enough of a weight difference to be noticed, and even influence humans? Suppose one wishes to achieve a similar reduction in weight simply by climbing up a vertical distance H above Earth's surface to reduce Earth's gravitational field by 2.5×10^{-7} g. We have

$$\frac{GM_e}{R^2} - \frac{GM_e}{(R+H)^2} = \frac{2GM_e H}{R^3} = g\frac{2H}{R} = 2.5\times10^{-7}\,g \qquad (15)$$

Solving for H, we obtain $H = 0.79$ m, or about five steps up. (Note that this results is in fact identical to the value Δh obtained previously, c.f. discussion following Eq.(7)). Hence going to a bedroom upstairs results in a weight change that is about 5 times larger than that produced by the tidal mechanism during a spring tide.

Another factor that one might wish to consider here is that the weight of the atmosphere overhead, and hence the atmospheric pressure on a human body, is reduced too. Such gravitational reactions of the atmosphere to the tidal forces are rather small,

$$\Delta P = 1.7\times10^{-7}\times1 \text{ atm} = 0.02 \text{ Pa}, \qquad (16)$$

but there are larger tides in the atmosphere caused by a radiational energy input from the Sun. Analysis of tides in atmosphere is difficult due to contamination by broad-band noise of meteorological origin; nevertheless the tidal changes in the atmospheric pressure are found to be as high as $\pm$ 3 Pa, and a human body would stretch by some 10^{-9} m. One should keep in mind that such a pressure change could be alternatively achieved climbing up 0.2 meters, and that purely meteorological pressure variations can be as high as $\pm$ 3000 Pa.

Other Effects

What happens when the Moon is at the perigee, i.e. at the point closest to Earth? While it is true that the Moon's pull on Earth is at its maximum, this circumstance *per se* has no bearing on the tidal mechanism. It is the difference in the Moon's pulls, described by Eq.(4), that creates the tidal force. Since the eccentricity of the Moon's orbit is $\varepsilon = 0.0549$, the average distance d_m in Eq.(2) and Eq.(4) is replaced by the perigee distance of $d_p = d_m\times(1 - \varepsilon) = $ 363,000 km. This increases the Moon's differential pull of Eq.(4) by a factor of about $1/(1-0.0549)^3 = 1.18$, i.e.

$$\Delta a_{m,perigee} = 1.34\times10^{-7} \text{ g}, \qquad (17)$$

and the tidal action of the Moon is about 18 % stronger. If the Moon's passage through the perigee coincides with spring tides, the combined tidal effect of the Sun and the Moon is approximately 12% stronger than that described by Eq.(14). Such perigean spring tides happen approximately every 6.5 months. Also, because of a three-body effects in the Sun-Earth-Moon system, the lunar perigee distance can be actually as small as 356,4000 km, and the Moon's tidal effects can be up to 25% stronger than those at the average distance [11].

Earth's orbit around the Sun is also elliptical, and the Earth passes through the perihelion (point closest to the Sun) around January 3. The eccentricity of Earth's orbit is only $\varepsilon = $ 0.017, and at the perihelion the Sun's differential pull of Eq.(2) increases by about $1/(1-0.017)^3 = 1.052$, i.e. by some 5%. Since the time of perigean spring tide is different every year, it will eventually coincide with Earth's passage through perihelion. This will produce the highest tides in the entire tidal cycle. If such unusually strong tides happen to coincide with meteorological tides caused by a winter storm with low atmospheric pressure and onshore winds, then coastal flooding is a definite possibility.

Length of a Day

As we pointed out above, for the Earth covered with an ideal, uniform ocean, the surface of the water would form a spheroid, whose long (major) axis would point in a direction determined by the position of the Moon (more about in the next Section). As the Earth and oceans rotate beneath the spheroid, continents crash into the water bulge during a high tide, piling water up. It is important to remember that the water of the ocean rotates together with the Earth – it is only the *shape* of the water that remains fixed in space. As seen from a geocentric reference frame that rotates with the Earth, the tidal bulges appear as waves that circulate around the globe crashing into continents, thus imparting an impulse to the continents. (Keep in mind that a traveling wave transports both momentum and energy in the direction of motion). These inelastic collisions between the tidal bulges and continents cause a gradual decrease in the Earth's spinning rate. As a result, days are getting longer, by about 1.6 milliseconds per century. While it does not seem much, over a period of 1500 years the accumulated time difference equals 2 hours. In other words, over the period of 1500 years the Earth has rotated through 30 degrees more than it would if the angular velocity in the past had today's value. This is beautifully verified by the solar eclipse of January 14, 484 A.D., that would have been observed in southern Spain, if the Earth had spun at a today's rate, but was in fact observed in Greece, 30 degrees of longitude eastward. [12]

On a quite different time scale, an extrapolation back some 370 million years shows that a day then was about 22 hours long, and there were about 400 such short days in a year. This agrees with results of analysis of Devonian corals, that reveal a stunning periodicity in growth rings – there are about 400 daily growth rings in each annual set of rings.

Future of the Earth - Moon System

Since neither the Earth nor the water is perfectly elastic, the rotation of the Earth causes a delay in response of the ocean to the tidal force, and the spheroid is not exactly aligned with the direction toward the Moon, c.f. Figure 2. As a result, the high tide appears later than the time of highest moon.
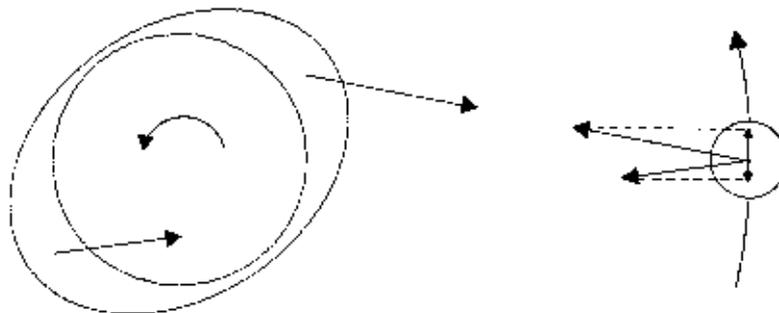
Figure 2. As the spinning Earth carries the water spheroid away, the spheroid is not
exactly aligned with the direction toward the Moon. The bulge closer to the Moon
exerts the gravitational force with larger component along the Moon's path than the

force exerted by the more distant bulge. This results in a non-zero net force in the direction of the Moon's orbital motion, that causes the Moon to move into a higher orbit and hence away from the Earth.

This misalignment of the water spheroid causes the net gravitational pull exerted by the water bulges to have a small forward component along the direction of the Moon motion. As a result, the Moon moves into a higher orbit and hence away from the Earth, at the rate of some 4 meters per 100 years, and the orbital period of the Moon increases. (*This is similar to launching a satellite from a Newton's Mountain by firing it horizontally from a cannon with a speed slightly higher than one required for a perfectly circular orbit. As a result, the satellite begins to follow an elliptical orbit, initially rising above the surface of the Earth while slowing down. Likewise, the forward component of the tidal force keeps" re-launching" the Moon ever higher, resulting in the Moon's recession from the Earth along a tightly wound spiral orbit. See Ref. [13] for a discussion of forces along the spiral trajectory.*) However, since the orbital period of the Moon increases at smaller rate than the length of the day does, both periods will eventually match. The Earth will be then tidally locked with the Moon, and the length of the day and the month will both be equal to some 50 present days, with the same side of Earth always facing the Moon. Note that the same side of the Moon already always faces the Earth, as the tidal action of the Earth on the Moon caused the Moon's original spin to slow down, and Moon became tidally locked with the Earth long time ago, in the sense that the Moon spins once on its axis for each revolution around the Earth.

Once the Earth becomes tidally locked with the Moon, the solar tides will tend to slow the Earth's rotation even more, so the day will be longer than the month and the Moon will rise in the West and set in the East. The water spheroid generated by the Sun will cause the high tide to appear earlier than the time of highest moon, a situation exactly opposite to that of Fig. 2. Then the tidal force of Earth on the Moon will pull the Moon into a lower orbit and eventually inside the Roche limit (18500 km), whereupon the Moon will disintegrate producing a ring around the Earth. [8]

Why is the water spheroid not aligned with the Moon?

A glance at Fig. 2 may suggest an explanation for the misalignment of the tidal spheroid and the Earth-Moon line and the resulting lag of a high tide with respect to the overhead position of the Moon. Due to the Earth's rotation, the speed of points on the Earth surface near the equator is about 1670 km/h. On the other hand, speed of very long waves on the surface of a relatively shallow water is determined by the Earth's gravity g and the depth of the ocean h, $v = \sqrt{gh}$. Assuming the average value for the oceans depth h = 4.0 km, one obtains v = 700 km/h. So a popular explanation claims that the traveling tidal wave simply can't move fast enough to remain directly under the Moon and ends up being carried away by Earth's rotation. Note that the speed of points along the Antarctic Circle at 66°33'39" S latitude is some 660 km/h, so the circulating tidal wave could actually keep up with the Earth-Moon line despite the rotation of the Earth.

Such an explanation ignores a crucial dynamic character of the periodic tide-rising forces that for a given location on the Earth has a period T = 12 h 25min. The observed tides are a steady-state response of the oscillating ocean to this external periodic driving force. For a simplest model for this response, imagine a hypothetical wide canal encircling the whole

Earth along the equator. The surface of the water in the canal has its own period of natural oscillations T_0, that could be estimated as a time required for a circulating tidal wave to travel along half of the globe, $T_0 = (20000 \text{ km})/(700 \text{ km/h}) = 29$ h. Clearly, the period of natural oscillations T_0 is much longer than the period of the driving force T, $T_0 \gg T$, and the theory of forced oscillations tells us that while the water surface will be forced to oscillate with the period T of the driving force, the response will lag behind the driving force by ½ of the period T, i.e. by about 6 hours. This means that the resulting tidal bulges in the ocean will be *perpendicular* to the Earth-Moon line [13], certainly a counterintuitive result.

Interestingly, the southern ocean is the only place on Earth where a circulating wave can travel practically unimpeded by land. For a channel encircling the Earth along the 65th parallel whose length is only 0.42 of the equatorial one, the period of natural oscillations of the water surface is some 12 h, which is close to the period of driving tidal force. The system is nearly resonant, and while the resulting forced oscillations will always have the period T, the lag of the response is now ¼ of the period, or about 3 h. Moving further south towards the Antarctic Circle one encounters progressively shorter canals for which the period of natural oscillations T_0 may become shorter than T. Under such conditions, the forced oscillations of the ocean still have the period T of the driving tidal force, but there is no time lag, i.e. one has a direct tide. Since the actual periods of natural oscillations of ocean in these locations may differ from our simple estimates, the value of latitude for which one would have a resonant response is difficult to determine theoretically. It is therefore interesting to note that the tidal lag in southern ocean along 65th parallel is about 2 h, and further south one observes a direct tide.

Falling into a Black Hole

We have seen that the tidal force manifests itself as downward compressional force toward the center of the Earth at point L and a stretching force pulling upward away from the center of the Earth at points C and F. This results in a permanent tidal flattening of Earth's poles since compressional tidal forces of Moon and Sun always add at Earth's poles. This tidal flattening of the poles occurs independent of the rotation of the Earth. Likewise, tidal forces generated by all planets add to the flattening of the Sun's poles. Most of the stars in our galaxy contribute to the tidal force that helps to flatten the galaxy.

What would be a fate of a person who were to fall feet-first into a black hole? That person would not feel the force of gravity, because of being in a free fall (similar to a person in a freely falling elevator or astronaut in orbiting shuttle not feeling her own weight), but would do feel a devastating tidal forces pulling upwards on her head and downwards on her feet, and squeezing her waistline inward [14].

Acknowledgments

References

1. *Miami Herald,* July 2, 1996.

2. I would like to use this opportunity and repeat my appeal to readers (c.f. *The Physics Teacher* **34**, p. 525, November 1996), to send me any example of a bad physics they might come across in newspapers or magazines. At the same time I would like to thank all my to-date contributors for their valuable input.

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